DYNAMIC MODELLING OF FLEXIBLE PAYLOADS MANIPULATED BY A SMART GRIPPER IN ROBOTIC ASSEMBLY

E.J. Park* and J.K. Mills**

Abstract

During robotic assembly of flexible payloads, gravity and inertial forces acting on these payloads may induce both static shape deformation and vibration of the thin-walled payloads. Static deformations, which arise from deformation of the part due to its own weight under the influence of gravity, lead to misalignment of mating points of the part. Unwanted vibrations, arising from inertial forces acting on the part as it is positioned for assembly, must be damped out before it can be mated with other parts. This paper investigates the development of dynamic models of arbitrarily shaped flexible thin-walled payloads grasped by a smart gripper, comprised of multiple linearly actuated fingers with non-contact proximity sensors. Such an actuated gripper allows both part-reshaping and active damping of unwanted vibrations of the part. Finite element modelling techniques are used to generate dynamic models of these arbitrarily shaped parts. Component mode synthesis methods are used to combine the dynamics of the actuated gripper with the dynamics of the thin-walled parts. The resultant dynamic model, developed with finite element modelling, is of high order, and hence model order reduction methods are employed to reduce the model order but retain the essential dynamics of the thin-walled payload gripper system. Using a two time-scale modelling technique, an integrated closed-loop modal truncation and control design procedure is used to design a shape and vibration controller for thin-walled payload. Simulation results modelling thin-walled sheet metal parts manipulated by an industrial robot confirm the validity of the proposed modelling approach.

Key Words

Dynamic modelling of flexible payload, robotic assembly, reduced-order modelling, finite element method, component mode synthesis, integrated design

1. Introduction

Robotic assembly of flexible thin-walled parts and components is hampered due to the fact that such parts will deform under their own weight due to gravity and vibrate due to inertial forces acting on them as they are positioned for assembly. This structural flexibility complicates the proper alignment and mating of surfaces and contact points for successful assembly. Further, assembly operations are delayed until unwanted vibration decays. To address these issues, a “smart” gripper is proposed with actuated fingers under closed-loop control, which can be used to simultaneously suppress unwanted part vibration and reshape parts. This smart gripper, shown schematically in Fig. 1, is fixed to a robotic manipulator and comprises multiple “point” linearly actuated fingers and non-contact proximity sensors, signal processors, and controllers, to enable real-time part-reshaping and vibration suppression.

Figure 1. Fanuc S-110 robot with smart gripper manipulating a flexible payload.

A procedure for the development of a dynamic model of an arbitrarily shaped thin-walled flexible payload actuated by a smart gripper is proposed in this paper. Due to the arbitrary shape of the thin-walled payloads, the finite element (FE) modelling technique is used to develop...
dynamic models of the payload. However, to reduce the order of the inherently high-order FE model, a procedure for the development of a reduced-order dynamic model of the flexible payload, including the smart gripper actuated fingers, is formulated. The main contribution of this paper is the theoretical development of an accurate, reduced-order modelling strategy for thin-walled flexible payloads grasped by an actuated smart gripper, suitable for the synthesis of simultaneous vibration and static shape controllers.

The organization of this paper is as follows. Section 2 reviews the relevant literature. In Section 3 we present a component modal model of actuated flexible payloads by a smart gripper. Section 4 derives the overall system model that integrates the robot and smart gripper actuated fingers dynamics to the component modal model of the flexible payload. In Section 5 we develop a two time-scale model of the flexible payload based on the singular perturbation theory. An integrated closed-loop modal truncation and vibration control synthesis procedure is proposed in Section 6. Section 7 presents numerical simulations using the reduced-order model developed in the preceding section, to show its effectiveness. Section 8 concludes the paper.

2. Literature Review

Manufacturing using robotic assembly of parts has been addressed in considerable detail in the literature. Related to this general problem is the robotic assembly of flexible parts. With applications in various areas, including the assembly of automotive bodies (e.g., [1]), this problem has received considerable attention. Substantial efforts have been reported on various aspects of robot tasks involving manipulation and grasping of flexible payloads, including development of controllers that address manipulation and bending of sheet metal [2] or manipulation of deformable beams [3]. Mills and Ing [1] developed payload vibration dynamics models for the control of flexible payloads manipulated by robots. Other researchers [4] performed experiments involving deforming and transporting a spring. More recently, work has been presented on two or more robots grasping and manipulating a flexible object (e.g., [5]).

The investigation of control of robotic manipulation of flexible payloads is closely related to flexible structure control. Here the main focus has been on vibration suppression of flexible structures; the static shape control problem and the simultaneous vibration and static shape deformation control problem have received less attention. One proposed approach to address these control problems has been to design and implement a dynamic shape controller (e.g., [6]) in which the state vector is composed of position variables describing the thin-walled structure shape and corresponding velocity variables.

A part of the overall model reduction strategy used in this paper is the component mode synthesis (CMS) method, proposed to overcome the problems inherent in modelling structures that require a high-order FE model. The Craig-Bampton method [7], also referred to as the dynamic impedance based method, is most widely used. The dynamic impedance describes the dynamic behaviour of a flexible component using the superposition of the static behaviour of the boundary degrees of freedom (DOF) and clamped normal modes of vibration obtained by fixing the boundary DOF. The Craig-Bampton method approximates the dynamic impedance by reducing the number of clamped normal modes. In our work, the CMS method allows an explicit expression of the coupling dynamics between the flexible payload and the smart gripper actuated fingers, while reducing the order of the FE model. This method guarantees the correct structural static shape behaviour when quasi-steady forces are applied at the non-boundary (or interior) DOF. The quasi-steady modes can be obtained by solving the eigenproblem involving the redistributed stiffness matrix of shape-defining interior DOF obtained by the static matrix condensation technique.

3. Dynamic Modelling of Flexible Payloads

This section presents a methodology for dynamic modelling of an arbitrarily shaped thin-walled flexible payload grasped by a smart gripper with multiple discrete actuated fingers. Because the payload has an arbitrary shape, the FE method is used to determine the stiffness and inertia matrices for controller synthesis. The proposed modelling strategy is as follows. Starting with a high-order FE model of a flexible payload, first the component mode synthesis and static matrix condensation (SMC) techniques are combined to produce a new component mode set that includes vibration and quasi-static modes. This allows an accurate representation of both the transient vibration and quasi-steady shape deformation behaviour of the flexible payload, and also captures highly coupled dynamic effects amongst the actuated fingers of the smart gripper and the flexible payload. The FE model is then transformed into a component modal model using the component mode set. Utilizing the two time-scale behaviour of the vibration and quasi-static modes, the component modal model is partitioned into “fast” and “slow” dynamic subsystems, corresponding to fast vibration modes and slow quasi-static modes representing static shape deformation. To develop a reduced-order fast dynamic subsystem model for controller synthesis, an integrated closed-loop (vibration) modal truncation and control design procedure is developed. This accounts for the interaction between vibration modes and controller/estimator gains, leading to the identification and retention of fewer dominant vibration modes in closed-loop. A schematic of the proposed modelling strategy is illustrated in Fig. 2.

The assumptions inherent in the modelling process are:

- Vibrations of the thin-walled flexible parts are linear.
- Static shape deformation of the part is linear.
- Each actuated finger represents a rigid grasp point on the part.
- The part, once grasped by the smart gripper, does not undergo rigid-body motion with respect to the smart gripper.

In the following, the actuated finger locations are chosen as the interface DOF by defining “actuator” constraint modes, utilizing the work of [8]. In our previous work [9], we found that the addition of quasi-static modes to
vibration and static correction modes can result in highly accurate simulation of behaviour of the flexible payloads, when subjected to both transient and quasi-steady external forces. The vibration modes contain correct frequency information when the frequency of external forcing is near or greater than the first natural frequency of the flexible payload, whereas the quasi-static modes bear correct frequency information when the frequencies of external applied forces (e.g., quasi-steady force) are much lower than the first natural frequency of the flexible payload.

The component modal equations of motion for an arbitrary flexible payload-smart gripper system are developed using the new CMS representation from our previous work [9]. Consider an arbitrary flexible payload grasped by actuated fingers as shown in Fig. 1. The undamped linear dynamic FE model of the flexible payload, subject to small deformations, is given by:

\[ M \ddot{q} + Kq = f \]  

where \( q \in \mathbb{R}^{n \times 1} \) is the vector of nodal coordinates, \( M \in \mathbb{R}^{n \times n} \) is the mass matrix, \( K \in \mathbb{R}^{n \times n} \) is the stiffness matrix, and \( f \in \mathbb{R}^{n \times 1} \) is the vector of external forces. Note that the order of the FE model given by (1) is typically high, and hence its dimensions must be reduced for controller synthesis.

The FE model in (1) can be partitioned as follows:

\[
\begin{bmatrix}
M_{rr} & M_{ra} & M_{ri} \\
M_{ar} & M_{aa} & M_{ai} \\
M_{ir} & M_{ia} & M_{ii}
\end{bmatrix}
\begin{bmatrix}
\dot{q}_r \\
\dot{q}_a \\
\dot{q}_i
\end{bmatrix} +
\begin{bmatrix}
K_{rr} & K_{ra} & K_{ri} \\
K_{ar} & K_{aa} & K_{ai} \\
K_{ir} & K_{ia} & K_{ii}
\end{bmatrix}
\begin{bmatrix}
q_r \\
q_a \\
q_i
\end{bmatrix} =
\begin{bmatrix}
f_r \\
f_a \\
f_i
\end{bmatrix}  
\]

where \( q = [q_r^T \ q_a^T \ q_i^T]^T \) and \( q_i \) represent the interface and interior nodal coordinates, respectively; the interface coordinates are partitioned into constrained coordinates \( q_r \), which represent those interface coordinates grasped by the non-actuated fingers that provides the rigid-body support to the flexible payload, and actuator coordinates \( q_a \), which represent those interface coordinates grasped by the actuated fingers; \( f_r = [f_r^T \ f_a^T]^T \) and \( f_i \) are the external forces acting on \( q_r \) and \( q_i \), respectively. We then partition \( q_i \) as \( q_i = [q_{i1}^T \ q_{i2}^T]^T \), where \( q_{i1} \) contains the selected interior coordinates that are used to define the shape of the flexible payload and \( q_{i2} \) contains the remaining coordinates. In our work, \( q_{i1} \) corresponds to the coordinates normal to the flexible payload that are measured by the non-collocated laser proximity sensors. Using the SMC technique, a Guyan transformation matrix \([10], \ T_1 \), can be formed to redistribute the stiffness and inertial effects associated with the \( q_{i2} \) coordinates to \( q_{i1} \), that is:

\[ q_i = T_1 q_{i1} \]  

where:

\[ T_1 = \begin{bmatrix} I_{11} \\ T'_1 \end{bmatrix} = \begin{bmatrix} I_{11} \\ -K_{22}^{-1}K_{21} \end{bmatrix} \]

in which \( T'_1 = -K_{22}^{-1}K_{21} \) is obtained by partitioning the stiffness matrix \( K_{ii} \) in (2) according to \( q_{i1} \) and \( q_{i2} \) coordinates:

\[
K_{ii} = \begin{bmatrix} K_{11} & K_{12} \\ K_{21} & K_{22} \end{bmatrix}
\]

Substitution of (3) into (2) yields a reduced-order representation:

\[
\begin{bmatrix}
M_{rr} & M_{ra} & M_{ri} \\
M_{ar} & M_{aa} & M_{ai} \\
M_{ir} & M_{ia} & M_{ii}
\end{bmatrix}
\begin{bmatrix}
\dot{q}_r \\
\dot{q}_a \\
\dot{q}_i
\end{bmatrix} +
\begin{bmatrix}
K_{rr} & K_{ra} & K_{ri} \\
K_{ar} & K_{aa} & K_{ai} \\
K_{ir} & K_{ia} & K_{ii}
\end{bmatrix}
\begin{bmatrix}
q_r \\
q_a \\
q_i
\end{bmatrix} =
\begin{bmatrix}
f_r \\
f_a \\
f_i
\end{bmatrix}
\]

or, equivalently, \( M_1 \dot{q} + K_1 q = f_1 \), where:

\[
\begin{align*}
M_{ri1} &= T_1^T M_{ri} T_1, \\
M_{ai1} &= T_1^T M_{ai} T_1, \\
M_{ir1} &= T_1^T M_{ir} T_1, \\
K_{ri1} &= T_1^T K_{ri} T_1, \\
K_{ai1} &= T_1^T K_{ai} T_1, \\
K_{ir1} &= T_1^T K_{ir} T_1,
\end{align*}
\]

\[
f_{i1} = T_1^T f_i.
\]

In the following, the order of the dynamic model (5) is reduced using the CMS method.
3.1 Component Mode Set

Using the CMS method, the nodal coordinates, \( q = [q_a^T \ q_c^T \ q_n^T]^T \), are represented in terms of generalized coordinates, \( p \), with the following coordinate transformation:

\[
q \approx \dot{q} = Tp
\]

where \( \dot{q} \) represents the approximated nodal coordinates of \( q \); and the component mode transformation matrix \( T \) is a matrix of preselected component modes: actuator constraint modes \( \Psi_c \), fixed-actuator vibration modes \( \Phi_v \), and fixed-actuator quasi-static modes \( \Phi_q \).

**Actuator Constraint Modes.** An actuator constraint mode \( [8] \) is defined as the steady-state deflection shapes of the flexible payload obtained by applying a unit static displacement to one of the actuator coordinates, while holding the remaining interface coordinates, \( q_j \), fixed. The actuator constraint modes \( \Psi_c \) represent the static behaviour of the actuator coordinates \( q_j \) due to the concentrated actuator forces at these locations. These modes describe the correct local deformation effects on the flexible payload due to the actuator fingers, and are obtained from:

\[
\Psi_c = \begin{bmatrix}
0 \\
I_{na} \\
\Psi_c \\
-\mathbf{K}_{ii}^{-1}\mathbf{K}_{ia}
\end{bmatrix}
\]

\[\text{where } I_{na} \text{ is the identity matrix.}\]

**Fixed-Actuator Vibration Modes.** The \( M_{ii} \)-normalized fixed-actuator modal matrix \( \Phi_v \) is obtained from the solution of the eigenproblem:

\[
\Phi_v = \{\lambda \mathbf{K}_v - \mathbf{M}_v \}^{-1} \mathbf{F}_v
\]

\[\text{where } \mathbf{K}_v = \mathbf{K}_v - \mathbf{K}_{fi} \text{ is a diagonal matrix of natural frequencies } \omega_{ij}, j = 1, \ldots, n_c; n_v \text{ represents the number of retained fixed-actuator vibration modes.}\]

**Fixed-Actuator Quasi-Static Modes.** A fixed-actuator quasi-static modal matrix \( \Phi_q \) is obtained from the solution of the eigenproblem:

\[
\mathbf{K}_{ii} \Phi_q = \Phi_q \Omega_q
\]

\[\text{where } \Omega_q = \text{diag}(\omega_{qj}^2) \text{ is a diagonal matrix of quasi-static frequencies } \omega_{qj}, j = 1, \ldots, n_q; n_q \text{ represents the number of fixed-actuator quasi-static modes.}\]

Using (4), we define \( \Phi_q' = T \Phi_q' \). Each quasi-static mode is orthogonalized with respect to the fixed-actuator vibration modes using the Gram-Schmidt procedure:

\[
\Psi_{j}'' = \Psi_{j}' - \sum_{m=1}^{n_q-1} \phi_{m}^T \Psi_{j}' \phi_{m} \quad (j = 1, \ldots, n_q)
\]

Once \( \Psi_{j}'' \) is found, it is normalized with respect to the mass matrix \( M_{ii} \) so that \( \Psi_{j}' \mathbf{M}_{ii} \Psi_{j} = 1 \). Finally, the fixed-actuator quasi-static modal matrix in (9) is written as \( \Phi_q = [\Psi_1, \ldots, \Psi_{n_q}] \).

3.2 Component Model

Using the actuator constraint modes, fixed-actuator vibration modes, and fixed-actuator quasi-static modes described in Section 3.1, we rewrite (5) in a component modal form. Redefining \( q \) in (6) as \( q = [q_a^T \ q_c^T \ q_n^T]^T \), the coordinate transformation in (6) is given by:

\[
[q_a] = Tp
\]

\[\text{where:}\]

\[
T = \begin{bmatrix}
I_{na} & 0 & 0 \\
\Psi_c & \Phi_v & \Phi_q
\end{bmatrix}, \quad p = \begin{bmatrix}
\eta \\
\xi
\end{bmatrix}
\]

\[\text{in which } q_a \in \mathbb{R}^{na \times 1} \text{ is the vector of actuator coordinates where } n_a \text{ represents the number of actuators (or actuated fingers), } \eta \in \mathbb{R}^{na \times 1} \text{ vector of fixed-actuator vibration modal coordinates, and } \xi \in \mathbb{R}^{n_a \times 1} \text{ vector of fixed-actuator quasi-static modal coordinates, respectively. The component modal form of (5) can be generated using the relations } \mathbf{M}_v \mathbf{T}^T \mathbf{M}_v \mathbf{T}, \mathbf{K}_v \mathbf{T}^T \mathbf{K}_v \mathbf{T}, \text{ and } \mathbf{f}_v = \mathbf{T}^T \mathbf{f}_v \text{ where } \mathbf{M}_v, \mathbf{K}_v, \text{ and } \mathbf{f}_v \text{ are the reduced-order matrices of (5), eliminating the elements corresponding to the constrained coordinates, } q_f.\]

4. System Modelling

Here, the equations of motion of the actuated flexible payload undergoing smart gripper actuated finger and robot-induced motions are briefly outlined, with details in [11]. Consider a flexible payload manipulated by a robot as shown in Fig. 3. The motion of the flexible payload is a superposition of robot-induced rigid-body motion and small elastic motions of the payload. The rigid-body motion of the payload is represented by the motion of the body coordinate frame \( b-xyz \), defined at a node on the payload in its undeformed state, with respect to an inertial coordinate frame \( o-XYZ \). All other nodes are defined relative to this frame.

Let \( \theta \) be the robot joint coordinates. The \( 3 \times 1 \) position vector of node \( i \) with respect to the \( o-XYZ \) frame is given by:

\[
o_i(\theta, \varepsilon_i) = o_b(\theta) + R_{ob}(\theta)(b_i^0 + \varepsilon_i)
\]

where \( o_b(\theta) \) is the position vector of the \( b-xyz \) frame with respect to the \( o-XYZ \) frame; \( R_{ob}(\theta) \) is the \( 3 \times 3 \) rotational transformation matrix from the \( b-xyz \) frame to \( o-XYZ \) frame; \( b_i^0 \) is the position vector of node \( i \) in the undeformed
state measured in the \(b\)-\(xyz\) frame; and \(\varepsilon_i\) is the elastic deformation vector of node \(i\) in the \(b\)-\(xyz\) frame. The first time derivative of \(r_i\) is obtained by ignoring the elastic deformation term \(\varepsilon_i\), that is \(b\r_i + \varepsilon_i \approx b\r_i^0\):

\[
\dot{r}_i \equiv J_\theta(\theta) \dot{\theta} + R_{ob}(\theta) \dot{\varepsilon}_i
\]

where:

\[
J_\theta(\theta) = \frac{\partial \varepsilon_b(\theta)}{\partial \theta} + \frac{\partial R_{ob}(\theta)}{\partial \theta} r_i^0.
\]

The Lagrangian of the flexible thin-walled payload is

\[
L = T - U_\varepsilon - U_g,
\]

where \(T\) is the kinetic energy, \(U_\varepsilon\) is the potential energy due to elastic deformation, and \(U_g\) is the potential energy due to gravity. Evaluating the Lagrange equations of motion with respect to the elastic deformation coordinates \(\varepsilon\), for which \(\varepsilon\) can be written in terms of elemental nodal coordinates \(q\) by \(\varepsilon = B_{be} q\) and \(B_{be}\) is a constant transformation matrix whose elements are either zeros or ones, we obtain the dynamic equations of the payload:

\[
M_\varepsilon B_{be} \ddot{q} + K_\varepsilon B_{be} q = B_{be} B_{af} - g_\varepsilon(\theta) - M_{\varepsilon\varepsilon}(\theta) \dot{\theta} - C_\varepsilon(\theta) \dot{\theta}
\]

where:

\[
M_\varepsilon = \begin{bmatrix} m_1 & \cdots & m_n \end{bmatrix}, \quad g_\varepsilon(\theta) = \frac{\partial G_\varepsilon}{\partial \varepsilon},
\]

\[
G_\varepsilon = g(R_{ob}(\theta)) \sum_{i=1}^{n} m_i \varepsilon_i z,
\]

\[
M_{\varepsilon\varepsilon}(\theta) = \sum_{i=1}^{n} m_i R_{ob}^T(\theta) J_\theta(\theta),
\]

and

\[
C_\varepsilon(\theta) = \dot{M}_{\varepsilon\theta}(\theta) + \frac{\partial (\dot{\varepsilon}^T M_{\varepsilon\theta}(\theta))}{\partial \varepsilon} = \dot{M}_{\varepsilon\theta}(\theta).
\]

\(M_{\varepsilon\theta}(\theta)\) and \(C_\varepsilon(\theta)\) describe the coupling effects of the robot geometry on the payload, \(f_a\) is the force exerted by the actuated fingers with respect to the element coordinate frames, and \(B\) is the actuated finger influence matrix.

Fig. 4 illustrates the rack-and-pinion arrangement of the finger actuators used to damp vibration and reshape the flexible payload, with the dynamic equations given by:

\[
m_a \ddot{\delta} = \frac{1}{r} \tau - f_i
\]

where \(m_a = J_p/r^2 + m_i\), \(\delta\) is a vector of displacements of the actuated finger, \(m_i\), is the linear guide mass, \(J_p\) is the pinion inertia, and \(r\) is the pinion radius. The remaining terms are described in Fig. 4. Note that \(f_a = R_{\delta a} f_\delta\) where \(R_{\delta a}\) is the direction cosine matrix of the actuated finger forces from \(f_\delta\) to \(f_a\), and similarly, \(\delta = R_{\delta a} q_\delta = R_{\delta a} B^T q\).
Equation (17) represents the dynamic equations of motion of the flexible payload actuated by the smart gripper actuated fingers as the flexible payload undergoes robot-induced motions.

5. Two Time-Scale Modelling

In this section, only final results of two time-scale modelling [12] of (17)—the dynamic behaviour of the quasi-static modes and vibration modes—is presented, as the details are given in our previous paper [13]. First, we write the partitioned form of the system mass matrix \( M_s \), stiffness matrix \( K_s \), and external force \( f_s \) in (16) as:

\[
\begin{bmatrix}
M_{aa} & M_{ai} \\
M_{ia} & M_{ii}
\end{bmatrix},
\begin{bmatrix}
K_{aa} & K_{ai} \\
K_{ia} & K_{ii}
\end{bmatrix}, \quad \text{and} \quad \begin{bmatrix}
f_a \\
f_i
\end{bmatrix}
\]

respectively. Using these partitioned matrices the expanded form of (17) can be written as:

\[
\begin{bmatrix}
M_a & P_v^T & P_q^T \\
-P_v & I & 0 \\
-P_q & 0 & I
\end{bmatrix}
\begin{bmatrix}
\ddot{\mathbf{q}}_a \\
\dot{\mathbf{q}}_a \\
\dot{\mathbf{q}}_i
\end{bmatrix}
+ \begin{bmatrix}
K_a & 0 & 0 \\
0 & \Omega_s & \Lambda_s \\
0 & \Omega_i & \Omega_{ii}
\end{bmatrix}
\begin{bmatrix}
\mathbf{q}_a \\
\eta \\
\xi
\end{bmatrix}
= \begin{bmatrix}
\mathbf{f}_a + \Psi_c^T f_i \\
0 \\
0
\end{bmatrix} \tau + \begin{bmatrix}
\Phi_c^T f_i \\
\Phi_c^T f_i \\
\Phi_c^T f_i
\end{bmatrix} (18)
\]

where:

\[
M_a = M_{aa} + M_{ai} \Psi_c + \Psi_c^T (M_{ia} + M_{ii} \Psi_c),
\]

\[
K_a = K_{aa} + K_{ai} \Psi_c + \Psi_c^T (K_{ia} + K_{ii} \Psi_c),
\]

\[
P_v = -\Phi_c^T (M_{ia} + M_{ii} \Psi_c),
\]

\[
P_q = -\Phi_c^T (M_{ia} + M_{ii} \Psi_c),
\]

\[
\Lambda_v = \Phi_c^T K_{ii} \Phi_q,
\]

and

\[
\Lambda_q = \Phi_c^T K_{ii} \Phi_q.
\]

Defining a state vector as \( \mathbf{x} = [\mathbf{q}_a^T \quad \eta^T \quad \xi^T \quad \dot{\mathbf{q}}_a^T \quad \dot{\mathbf{q}}_i^T \quad \dot{\eta}^T \quad \dot{\xi}^T]^T \), (18) becomes:

\[
\dot{\mathbf{x}} = \mathbf{A} \mathbf{x} + \mathbf{B} \mathbf{u}
\]

(19)

where:

\[
\mathbf{E} = \begin{bmatrix}
I & 0 \\
0 & \mathbf{M}_i
\end{bmatrix}, \quad \mathbf{A'} = \begin{bmatrix}
0 & I \\
-K_v & 0
\end{bmatrix}, \quad \mathbf{B'} = \begin{bmatrix}
0 \\
\mathbf{B}_c
\end{bmatrix}, \quad \mathbf{u} = \tau; \quad \mathbf{A} = \mathbf{E}^{-1} \mathbf{A'}, \quad \mathbf{B} = \mathbf{E}^{-1} \mathbf{B'}.
\]

Equation (19) is written as a two time-scale dynamic system with the fast time-scale dynamics representing thin-walled payload vibration dynamics and the slow time-scale dynamics representing quasi-static shape deformation.

Defining \( \mu = 1/\sqrt{k} \), and factoring \( \Omega_k = (1/\mu) \Omega_k \), we can define a new variable \( \mathbf{z} \) as \( \mathbf{z} = \Omega_k \eta = \Omega_k \eta / \mu \). We introduce a fast time-scale as \( \tau = t/\mu \) and define the following fast time-scale variables: \( \mathbf{z} = \mathbf{z} - \tilde{\mathbf{z}} \), \( \dot{\mathbf{q}}_a = \tilde{\mathbf{q}}_a - \mathbf{q}_a \), \( \dot{\tau} = \tau - \tilde{\tau} \), \( \tilde{f}_a = f_a - \mathbf{f}_a \), and \( \mathbf{f}_i = f_i - \mathbf{f}_i \), where (\( \tilde{\cdot} \)) indicates that the corresponding quantities are evaluated in the fast time-scale and (\( \dot{\cdot} \)) indicates that the corresponding quantities are evaluated in the slow time-scale (i.e., \( \mu = 0 \)). Then, the fast and slow time-scale state vectors can be defined as \( \mathbf{x} = [\mathbf{q}_a^T \quad \mathbf{z}^T \quad \dot{\mathbf{q}}_a^T \quad (dz/\mu dt)^T]^T \) and \( \tilde{\mathbf{x}} = [\mathbf{q}_a^T \quad \dot{\mathbf{q}}_a^T \quad \dot{\mathbf{q}}_i^T \quad \dot{\mathbf{z}}]^T \), respectively. The resultant fast time-scale dynamic subsystem representing robot motion induced vibrations in the sheet metal part is written as:

\[
\begin{align*}
\dot{\mathbf{x}} &= \mathbf{A} \mathbf{x} + \mathbf{B} \mathbf{u} \\
\dot{\mathbf{y}} &= \mathbf{D} \mathbf{z}
\end{align*}
\]

(20)

where:

\[
\mathbf{A} = \begin{bmatrix}
0 & 0 & I & 0 \\
0 & 0 & 0 & I \\
-\mathbf{M} & \mathbf{P}_f^T & 0 & 0 \\
\mathbf{C}_1 \mathbf{\Phi}_c & \mathbf{C}_1 \mathbf{\Phi}_q \Omega_k^{-1} & 0 & 0
\end{bmatrix},
\]

\[
\mathbf{B} = \begin{bmatrix}
0 \\
\mathbf{C}_1 \mathbf{\Phi}_c \mathbf{\Omega}_k^{-1}
\end{bmatrix},
\]

\[
\mathbf{C} = \begin{bmatrix}
\mathbf{C}_a & 0 & 0 & 0 \\
\mathbf{C}_{11} \mathbf{\Phi}_c & \mathbf{C}_{11} \mathbf{\Phi}_q \mathbf{\Omega}_k & 0 & 0
\end{bmatrix}
\]

in which \( \mathbf{M} = (\mathbf{M}_a + \mathbf{P}_f^T \mathbf{P}_u)^{-1} \). The slow time-scale dynamic subsystem representing the quasi-static deformations, that is, static deformations caused by gravity, is given by:

\[
\begin{align*}
\dot{\mathbf{x}} &= \mathbf{A} \mathbf{x} + \mathbf{B} \mathbf{u} \\
\dot{\mathbf{y}} &= \mathbf{C} \mathbf{x} + \mathbf{D} \mathbf{z}
\end{align*}
\]

(21)

where:

\[
\mathbf{A} = \begin{bmatrix}
0 & 0 & I & 0 \\
0 & 0 & 0 & I \\
-\mathbf{M} & \mathbf{P}_f^T & 0 & 0 \\
\mathbf{C}_1 \mathbf{\Phi}_c & \mathbf{C}_1 \mathbf{\Phi}_q \Omega_k^{-1} & 0 & 0
\end{bmatrix},
\]

\[
\mathbf{B} = \begin{bmatrix}
0 \\
\mathbf{C}_1 \mathbf{\Phi}_c \mathbf{\Omega}_k^{-1}
\end{bmatrix},
\]

\[
\mathbf{C} = \begin{bmatrix}
\mathbf{C}_a & 0 & 0 & 0 \\
\mathbf{C}_{11} \mathbf{\Phi}_c & \mathbf{C}_{11} \mathbf{\Phi}_q \mathbf{\Omega}_k & 0 & 0
\end{bmatrix}
\]
6. Integrated Design

In this section, we present an integrated design procedure that determines a reduced-order component modal model (ROM) and its low-order controller simultaneously and iteratively. The advantage of the proposed integrated design strategy is that the interaction between vibration modes and controller/estimator gains can be accounted for, leading to the identification and retention of a minimum number of dominant vibration modes in closed loop. A reduced-order component model provides better closed-loop accuracy than the typical normal-vibration-mode-based reduced-order model due to the preservation of structure and actuator interaction [14]. Determination of the number of vibration modes to retain has traditionally been achieved by trial and error or an open-loop modal truncation procedure [15].

Fig. 5 illustrates the approach used in our work to determine a low-order controller from a full-order model. The objective of the proposed integrated design procedure is to find a ROM and its low-order controller simultaneously. In this way, the interaction between vibration modes and controller can be accounted for, leading to the identification and retention of fewer dominant vibration modes that meet a given set of closed-loop performance specifications, resulting in a minimum-order vibration controller.

Figure 5. Basic approaches to low-order control design.

As shown in Fig. 6, first the dominant (fixed-interface) vibration modes of the full-order component modal model are ranked in decreasing value in open-loop using the Hankel singular values of the observability and controllability matrices [15]. Next, the proposed integrated design procedure is used to simultaneously find the minimum number of closed-loop dominant vibration modes to be retained, incorporating the Hankel singular values, and LQG compensator gains.

6.1 Open-Loop Modal Truncation Strategy

The dynamics of the vibration modal coordinates in the fast subsystem is:

\[
\Omega_k^{-1} \frac{d^2 \tilde{z}}{dt^2} = -\tilde{z} + P_c \tilde{q}_v
\]

\[
\tilde{y}_z = C_v \Phi_v \Omega_v^{-1} \tilde{z}
\]

(22)

where

\[
M = (M_a + P_q^T P_q)^{-1}.
\]

Using the open-loop Hankel norm modal truncation error to quantitatively determine the \(n_v\) retained dominant vibration modes, we identify open-loop dominant vibration modes and rank them among the \(n_f\) vibration modes, obtained from the full-order plant. The open-loop Hankel norm truncation error is defined as:

\[
e_h = ||P - P_r||_h
\]

(24)

where \(P_r\) is the open-loop reduced-order plant with \(n_v\) retained vibration modes. \(||P - P_r||_h\) is the largest Hankel norm of truncated modes, that is:

\[
||P - P_r||_h = \max \ ||P_i||_h \quad (i = n_v + 1, \ldots, n_f)
\]

(25)

where \(||P_i||_h = \gamma_i = \sqrt{\text{eig}_i(W_c W_o)}\). Here, \(\gamma_i\) is called the Hankel singular value of the \(i\)-th norm; \(\text{eig}_i(\cdot)\) denotes the
eigenvalue of the $i$-th norm; and $W_c$ and $W_s$ are the controllability and observability grammians, corresponding to actuator and sensor locations, respectively. The dominant vibration modes are identified as vibration modes with the largest $\gamma_i$. In the next subsection, these Hankel singular values are used to form a closed-loop modal truncation index.

### 6.2 Closed-Loop Integrated Design

To design the compensator gains, a performance index $J_v$ is defined to first determine the solution to the fast LQR problem:

$$J_v = \frac{1}{2} \int_0^T (\mathbf{x}^T \mathbf{Q} \mathbf{x} + \mathbf{u}^T \mathbf{R} \mathbf{u}) dt \quad (26)$$

where $\mathbf{x}$ is the estimated fast state and $\mathbf{Q}$ and $\mathbf{R}$ are weighting matrices. To minimize the performance index, an optimal state feedback control is used, that is:

$$\mathbf{u} = -\mathbf{G} \mathbf{x} \quad (27)$$

with the fast feedback control gain matrix $\mathbf{G} = -\mathbf{R}^{-1} \mathbf{B}^T \mathbf{S}_c$ and fast (bias-free) estimator gain matrix $\mathbf{L} = \mathbf{S}_c \mathbf{C}^T \mathbf{V}^{-1}$. Here $\mathbf{S}_c$ and $\mathbf{S}_e$ are the positive-definite solutions to a steady-state controller and estimator algebraic Riccati equations, with details omitted from this paper.

To determine $\mathbf{Q}$ and $\mathbf{R}$ in (26) we choose the weights a priori to yield meaningful and consistent physical quantities in the performance cost, $J_v$ [16]. For example, the vibration control problem can be considered as a minimization of the system energy [17]:

$$\mathbf{Q} = \begin{bmatrix} \alpha \mathbf{K}_c & 0 \\ 0 & \beta \mathbf{M}_c \end{bmatrix} \quad \text{and} \quad \mathbf{R} = \begin{bmatrix} \tilde{\mathbf{B}}_c^T \mathbf{K}_c^{-1} \tilde{\mathbf{B}}_c \\ \mathbf{K}_c \end{bmatrix} \quad (28)$$

where the coefficients $\alpha$ and $\beta$ weight the flexible payload’s strain and kinetic energy in relation to its vibration behaviour. $\mathbf{K}_c$, $\mathbf{M}_c$, and $\tilde{\mathbf{B}}_c$ are the stiffness, mass, and control input matrices, respectively, corresponding to the fast vibration subsystem:

$$\mathbf{K}_c = \begin{bmatrix} \mathbf{K}_a & 0 \\ 0 & \Omega_k \end{bmatrix}, \quad \mathbf{M}_c = \begin{bmatrix} \mathbf{M}_a & \mathbf{P}_v \Omega_v^{-1} \\ -\Omega_v \mathbf{P}_v & \mathbf{I} \end{bmatrix}, \quad \text{and} \quad \tilde{\mathbf{B}}_c = \begin{bmatrix} \mathbf{B}_a \\ 0 \end{bmatrix}$$

With the above definitions, the LQG compensator is implicitly weighted by $\alpha$ and $\beta$ such that the performance cost $J_v$ provides a measure of the fast vibration subsystem strain and kinetic energy.

**Performance Specification.** Let $\rho_j$ define the $j$-th performance specification, and let $a_j$ quantify the performance. To approximate the full-order plant $\mathbf{P}$ by a reduced-order plant $\mathbf{P}_r$, the approximation should have the following properties:

**Performance:** The closed-loop response obtained when the low-order compensator is applied to the full-order plant $\mathbf{P}$ should result in steady-state measurement error within a pre-specified value (i.e., 0.5mm):

$$\rho_1 \equiv \max_j |e_{ssj}| \leq a_1 = 0.5 \quad (j = 1, \ldots, n_s) \quad (29)$$

where $n_s$ is the number of proximity sensors and $e_{ss}$ is the steady-state measurement error:

$$e_{ss} = \lim_{t \to \infty} (y_i - q^d_i) \quad (30)$$

The 0.5mm requirement is the typical assembly tolerance for automotive sheet metal parts.

**Robustness:** The low-order compensator, which stabilizes the reduced-order closed-loop matrix $\mathbf{H}_r = \mathbf{A} + \tilde{\mathbf{B}} \mathbf{G} + \mathbf{L} \mathbf{C}$, should also stabilize the full-order plant $\mathbf{P}$. To quantify this constraint, let the truncated (or residual) plant $\mathbf{P}_r$ be an additive perturbation to the reduced-order plant $\mathbf{P}_r$, that is, $\mathbf{P}_r = \mathbf{P} - \mathbf{P}_r$ due to the truncated vibration modes. Then, this perturbation is bounded by [18]:

$$||\mathbf{P}_r||_\infty \leq 2 \sum_{i=n_s+1}^{n_f} \gamma_i = D \quad (31)$$

where $|| \cdot ||_\infty$ is the $H_\infty$ norm and $D$ is an additive perturbation bound. Then, according to the Small Gain Theorem, the stable closed-loop system $\mathbf{H}_r$ is guaranteed to be stable if and only if the following closed-loop robustness specification holds [18]:

$$\rho_2 \equiv ||\mathbf{H}_r||_\infty \leq a_2 \equiv \frac{1}{D} \quad (32)$$

In the above performance specification equations, $a_1$ and $a_2$ denote real specification values, which bound the truncated (or residual) vibration mode dynamics. Here, it is assumed that the perturbation due to the truncated vibration modes is the dominant perturbation to the reduced-order plant dynamics. The remaining part of this section presents the integrated design procedure to define the upper-bound of model uncertainty due to the closed-loop modal truncation during the control design phase.

**Optimization Problem.** Here, the optimization problem for achieving the integrated design goal is formulated. The following closed-loop modal truncation index, representing the significance of each vibration mode in closed loop is [15]:

$$\sigma_i = \gamma_i \lambda_i \quad (33)$$

where $\sigma_i = \gamma_i \lambda_i$ is a measure of the $i$-th pole mobility, a measure of the controllability and observability of the $i$-th vibration mode ($\gamma_i$) coupled to the $i$-th LQG characteristic value ($\lambda_i$). The first objective function, $f_1$, is the maximum value of the pole mobility of the flexible payload, given by:
This measure allows the identification of dominant vibration modes in closed-loop for a given (balanced) LQG compensator. It is well known that the performance index $J_v$ in (26) can be expressed in terms of initial states [16], that is, $\hat{x}_o = \hat{x} (t = 0)$:

$$J_v = \frac{1}{2} \int_0^T (\hat{x}^T \hat{Q} \hat{x} + \hat{u}^T \hat{R} \hat{u}) \, dt = \frac{1}{2} \hat{x}_o^T S \hat{x}_o$$  \hspace{1cm} (35)$$

where $S$ is the positive-definite solution to the steady-state controller and estimator algebraic Riccati equations, which was derived in our previous work [19]:

$$S = \begin{bmatrix} 2\zeta M_t & \alpha_1 \dot{M}_c \\ \alpha_1 \dot{M}_c & \zeta \dot{M}_t K_c^{-1} M_t \end{bmatrix}$$  \hspace{1cm} (36)$$

where $\alpha_1 = \sqrt{1 + \alpha - 1}$, $\zeta = \sqrt{2\alpha + \beta}$ and $\dot{M}_t = \dot{M}_c^{1/2} (\dot{M}_c^{-1/2} K_c M_c^{-1/2})^{1/2} M_t^{1/2}$.

Then, the minimum value of the performance index $J_v$ can be taken as the second objective function:

$$f_2 = \hat{x}_o^T S \hat{x}_o$$  \hspace{1cm} (37)$$

which minimizes control energy for vibration suppression. The initial states in (37) may be approximated from open-loop response of the flexible payload, when excited by external disturbances. In [19], an approximate measure of the initial states was derived based on the work of [17]:

$$\hat{x}_o = \begin{bmatrix} \hat{p} \\ \hat{\dot{p}} \end{bmatrix} \approx \begin{bmatrix} K_c^{-1} B_c M_c^{1/2} \\ K_c^{-1/2} B_c M_c^{1/2} \end{bmatrix}$$  \hspace{1cm} (38)$$

Finally, substitution of (36) and (39) into (35) gives the approximate measure of the performance index and the objective function, $f_2$:

$$J_v \approx \hat{J}_v (\alpha, \beta) = \begin{bmatrix} K_c^{-1} B_c M_c^{1/2} \\ K_c^{-1/2} B_c M_c^{1/2} \end{bmatrix}^T \begin{bmatrix} 2\zeta M_t & \alpha_1 \dot{M}_c \\ \alpha_1 \dot{M}_c & \zeta \dot{M}_t K_c^{-1} M_t \end{bmatrix} \begin{bmatrix} K_c^{-1} B_c M_c^{1/2} \\ K_c^{-1/2} B_c M_c^{1/2} \end{bmatrix}$$  \hspace{1cm} (39)$$

Therefore, the second objective function is the minimization of $J_v$, which represents strain and kinetic energy and the quasi-steady “work” of the controller associated with the fast vibration behaviour of the flexible payload.

In summary, the integrated design optimization problem can be expressed as:

$$\max f_1 (\min f_2) \text{ subject to } p_1 \leq a_1 \text{ and } p_2 \leq a_2$$

for $n_v$, which represents the number of retained (fast time-scale) fixed-actuator vibration modes. Solving the above optimization problem (also outlined in Fig. 6), closed-loop modal truncation and minimum-order balanced LQG compensator gains are obtained simultaneously.

### 6.3 Two Time-Scale Controller Design

The closed-loop controller proposed for the fast and slow time-scale subsystems is a composite modal controller [12] given as:

$$u = \bar{u}(q_\alpha, \tilde{\xi}, \tilde{q}_\alpha, \dot{\tilde{q}}_\alpha) + \bar{u} \left( q_\alpha, \dot{\tilde{q}}_\alpha, \frac{d\tilde{q}}{dt} \right)$$  \hspace{1cm} (40)$$

A schematic of the two time-scale control and estimation [13] is shown in Fig. 7. Note that the formulation of this controller (along with its experimental demonstration) is a contribution of [13], and not the focus of the current paper, which discusses reduced-order modelling issues.

Figure 7. Two time-scale control and estimation.
7. Numerical Simulations

Modelling a Fanuc S-110R robot (which was shown schematically in Fig. 1; its Denavit-Hartenberg parameters are given in Table 1), the motion of a rectangular thin-walled sheet metal part grasped by the smart gripper actuated fingers, undergoing a large rigid-body motion, as shown in Fig. 8, was simulated using MATLAB®.

### Table 1
**FANUC S-110R Denavit-Hartenberg Parameters**

<table>
<thead>
<tr>
<th>Link</th>
<th>Twist $\alpha_i$ (deg)</th>
<th>Length $l_i$ (m)</th>
<th>Offset $d_i$ (m)</th>
<th>Angle Range $\theta_i$ (deg)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>90</td>
<td>0</td>
<td>0.780</td>
<td>$\pm 150$</td>
</tr>
<tr>
<td>2</td>
<td>0</td>
<td>0.750</td>
<td>0</td>
<td>$-40 \rightarrow +45$</td>
</tr>
<tr>
<td>3</td>
<td>0</td>
<td>0.900</td>
<td>0</td>
<td>$-45 \rightarrow +20$</td>
</tr>
<tr>
<td>4</td>
<td>9</td>
<td>0</td>
<td>0.105</td>
<td>190</td>
</tr>
<tr>
<td>5</td>
<td>$-90$</td>
<td>0</td>
<td>0.100</td>
<td>360</td>
</tr>
<tr>
<td>6</td>
<td>0</td>
<td>0</td>
<td>0.100</td>
<td>340</td>
</tr>
</tbody>
</table>

Figure 8. Rigid-body motion configuration of sheet metal plate (o—joints; •—nodes).

The model-order reduction strategy proposed in this paper was applied to this payload. The physical parameters of the ANSI 304 steel thin-walled sheet metal payload are: length = 1.0 m, width = 0.8 m, thickness = 0.001 m, modulus of elasticity = 197 GPa, density = 8030 kg/m³. This payload was discretized via the FE method using thin-plate elements, as shown in Fig. 9. Table 2 summarizes the vibration and quasi-static modal parameters. Each node has one out-of-plane displacement and two rotational degrees of freedom. Node 13 at the plate’s centre was constrained to prevent the payload from undergoing rigid-body motion with respect to the robot’s end-effector. The smart gripper actuated fingers are placed at nodes 7, 9, 17, and 19. The non-collocated (non-contact) proximity sensors are placed at the four corners of the plate: nodes 1, 5, 21, and 25, where maximum deflections occur. Damping effects were neglected in this simulation study.

### Table 2
**Vibration and Quasi-Static Modal Parameters (Hz)**

<table>
<thead>
<tr>
<th>Mode No.</th>
<th>Fixed-Actuator Vibration Modes</th>
<th>Fixed-Actuator Quasi-Static Modes</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>5.8</td>
<td>0.31</td>
</tr>
<tr>
<td>2</td>
<td>8.4</td>
<td>0.41</td>
</tr>
<tr>
<td>3</td>
<td>9.7</td>
<td>0.43</td>
</tr>
<tr>
<td>4</td>
<td>9.9</td>
<td>0.45</td>
</tr>
<tr>
<td>5</td>
<td>11.5</td>
<td>–</td>
</tr>
<tr>
<td>6</td>
<td>17.4</td>
<td>–</td>
</tr>
<tr>
<td>7</td>
<td>21.3</td>
<td>–</td>
</tr>
<tr>
<td>8</td>
<td>28.3</td>
<td>–</td>
</tr>
</tbody>
</table>

Figure 9. Finite element mesh on sheet metal plate, b-xyz = body coordinate frame (o—noncollocated sensor placements; •—collocated actuated finger and sensor placements).

Once the 72nd-order FE model of the payload was computed, the reduced-order modelling strategy was applied. First, the component modal model of the sheet metal plate and the equations of motion of the overall system were obtained. Note that the four out-of-plane coordinates at nodes 7, 9, 17, and 19 were selected as the interior coordinates that define the quasi-static shape of the plate, resulting in four fixed-actuator quasi-static modes. The stiffness and inertial effects of the other coordinates were redistributed to these selected coordinates in computing the fixed-actuator quasi-static modes. Second, two time-scale modelling was carried out on this system model. Next, using the open-loop modal truncation procedure (in Section 6.1), the Hankel singular values $\gamma_i$ for the first five fixed-actuator vibration modes were computed, given in Table 3. It can be seen that the Hankel singular value of the fourth fixed-actuator vibration mode is zero, implying that it is uncontrollable. As shown in Table 3, the Hankel singular values show that the first three fixed-actuator vibration modes are dominant in open loop. The next step was to apply the closed-loop integrated design procedure to obtain the minimum number of dominant fixed-actuator vibration modes and the balanced LQG compensator. Using the linear programming software MATLAB® Optimization Toolbox to find weights for the objective...
Table 3
Vibration Modal Truncation Parameters

<table>
<thead>
<tr>
<th>Mode</th>
<th>Freq (Hz)</th>
<th>$\gamma_i$</th>
<th>$\lambda_i \times 10^4$</th>
<th>$\sigma_i \times 10^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>5.8</td>
<td>0.037</td>
<td>1.60</td>
<td>5.9</td>
</tr>
<tr>
<td>2</td>
<td>8.4</td>
<td>0.029</td>
<td>1.57</td>
<td>4.5</td>
</tr>
<tr>
<td>3</td>
<td>9.7</td>
<td>0.019</td>
<td>1.69</td>
<td>3.2</td>
</tr>
<tr>
<td>4</td>
<td>9.9</td>
<td>0.0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>11.5</td>
<td>0.002</td>
<td>1.62</td>
<td>0.32</td>
</tr>
</tbody>
</table>

Table 4
Closed-Loop Fixed-Actuator Vibration Modal Truncation Results

<table>
<thead>
<tr>
<th>Design Spec.</th>
<th>Vibration Modes</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>$\rho_1$</td>
<td>0.36 mm</td>
</tr>
<tr>
<td>$\alpha_1$</td>
<td>0.5 mm</td>
</tr>
<tr>
<td>$\rho_1 \leq \alpha_1$?</td>
<td>Yes</td>
</tr>
<tr>
<td>$\rho_2$</td>
<td>3.3</td>
</tr>
<tr>
<td>$\alpha_2$</td>
<td>13</td>
</tr>
<tr>
<td>$\rho_2 \leq \alpha_2$?</td>
<td>Yes</td>
</tr>
</tbody>
</table>

function $f_2$ in (39), the balanced LQG characteristic values ($\lambda_i$) were obtained and are shown in Table 3. The closed-loop modal truncation index ($\sigma_i$) that measures the closed-loop pole mobility of the $i$-th vibration mode were also computed and are shown in the last column of Table 3. Starting with the least dominant vibration mode in closed-loop—that is, the vibration mode with the smallest $\sigma_i$—the vibration modes are truncated one by one using the closed-loop modal truncation procedure, as outlined in Fig. 6. Table 4 shows the closed-loop modal truncation results. Notice that at least three modes ($n^*$) are needed in the control design to satisfy the design specifications $\rho_1$ and $\rho_2$.

The open-loop deflection profile at node 1 is shown in Fig. 10. This plot is shown in this paper to demonstrate the effectiveness of the new component mode set that includes the fixed-actuator quasi-static modes, in simulating open-loop dynamic response of the flexible payload subject to both gravity and robot motion-induced forces. The out-of-plane deflection at node 1, using only eight fixed-actuator vibration modes (noted as 8V), four fixed-actuator vibration modes, and four fixed-actuator quasi-static modes (noted as 4V + 4Q), in response to the rigid-body motion is shown in Fig. 10. Note that the two modal approximations, 8V and 4V + 4Q, are constructed with the same number of modes. It can be concluded from the figure that both 8V and 4V + 4Q closely match the exact deflection plot, which was obtained from the full-order FE model. For a clearer comparison between the two modal approximations, Fig. 11 shows their deviations from the exact deflection plot. Fig. 11, which compares the R.M.S. norm errors [9] of the two modal approximations over all nodes, shows that 4V + 4Q gives better result. This is due to the ability of the fixed-actuator quasi-static modes to account for the gravity effects.

Fig. 12 shows the closed-loop deflection profile at node 1. The state-space representation of the reduced-order model, which was obtained using the above results (i.e., three closed-loop dominant vibration modes and four quasi-static modes), was employed to compute the feedback control gains. The control gains were computed using the composite modal controller and the separate-bias Kalman estimator [13] in Fig. 7. In Fig. 12, the solid line represents the closed-loop result and the dashed line represents the open-loop result, which also corresponds to the 4V + 4Q line in Fig. 10. The feedback control was applied at 0.4 sec, meaning that the actuated fingers were commanded in closed-loop after this time. It was observed from this simulation result that the static shape deformation of $-5$ mm was corrected quickly and the vibration in the sheet metal part was damped out successfully, in comparison to the open-loop. These results verify the effectiveness of the proposed reduced-modelling strategy for the simultaneous vibration and static shape control. Finally, Fig. 13 shows a picture of a prototype smart gripper built for an automotive fender.
8. Conclusion

Robotic assembly of flexible thin-walled payloads is hampered due to the fact that the payloads will vibrate due to inertial forces acting on them and deform under their own weight due to gravity as they are positioned for successful assembly. In order to solve this problem, a smart gripper with actuated fingers is proposed that can be used to simultaneously damp out unwanted payload vibrations and reshape distorted payloads in closed-loop control. This paper focuses on the development of a dynamic model of arbitrarily shaped flexible thin-walled payloads grasped by the smart gripper. The main contribution of this paper lies in the theoretical development of a model-order reduction procedure, which can be used to facilitate the design of a simultaneous vibration and static shape controller for such flexible payloads. The proposed model-order reduction procedure, which retains essential dynamics of the thin-walled payload gripper system, combines the finite element modelling technique, component mode synthesis method, the two time-scale modelling technique, and an integrated closed-modal truncation/control design procedure. Simulation results modelling an automotive sheet metal plate and industrial manipulator verify the validity of the proposed modelling approach.

References

Biographies

Edward J. Park received his B.A.Sc. degree from the University of British Columbia, Vancouver, Canada, in 1996 and his M.A.Sc. and Ph.D. degrees from the University of Toronto, Canada, in 1999 and 2003, respectively, all in mechanical engineering. Since 2003 he has been with the Department of Mechanical Engineering, University of Victoria, Canada, where he is currently an assistant professor. His research interests include robotics, dynamics and control, sensors and actuators, neuroprosthetics, and biomechanics.

James K. Mills received his B.A.Sc. degree from the University of Manitoba, Winnipeg, Canada, and his M.A.Sc. degree from the University of Toronto, Canada, in 1980 and 1982, respectively, both in electrical engineering. After working in a consulting company to assist in the development of a low-cost inertial attitude and heading reference unit for use in an underwater tow-body, he returned to the University of Toronto and received his Ph.D. in mechanical engineering in 1987. Immediately thereafter he joined DSMA International Ltd., where he worked on the conceptualization of a number of projects related to aircraft wind-tunnel testing. Since 1988 he has been with the Department of Mechanical and Industrial Engineering, University of Toronto, where he is currently a professor. His research interests have encompassed a number of related areas, including robot control, control of multirobots, control of flexible link robots, design of actuators, localization, development of fixtureless assembly technology, design and control of high-speed machines, and development of neural network controllers. He has published over 250 journal and conference papers and supervised over 50 masters and Ph.D. students and a number of post-doctoral fellows and research engineers. He was an invited visiting professor at the Centre for Artificial Intelligence and Robotics in Bangalore, India, and the University of Science and Technology in Hong Kong as well as the Chinese University of Hong Kong (the latter during 2002–03). He has also completed a three-year term as associate chair of the Department of Mechanical and Industrial Engineering and has made significant advances in the development of a new mechatronics curriculum and funding for that programme.