This article presents a technical demonstration of a system for determining the three-dimensional spatial location of complexly shaped, thin-walled sheet metal parts grasped by robots during assembly. For successful part assembly, the precise location of grasped parts (essential for successful mating of parts) must be achieved. A localization system is implemented to determine the accurate position and orientation of a sheet metal part that has been picked up by a robot from an arbitrary location. The proposed localization system employs a novel sensing method, utilizing laser-based proximity and edge detectors, to extract the part feature data in real time. These geometrical feature data are incorporated into an existing localization algorithm, which is based on the singular value decomposition formulation of the part localization problem. The sensing method is particularly effective in measuring 3-D feature geometry (i.e., thin edges) of sheet metal parts. An experimental single-robot test bed has been developed to demonstrate the feasibility of the part localization concept for a single sheet metal part. The experimental results obtained from the test bed demonstrate that the system can be effectively used for the localization of thin-walled sheet metal parts.

1. INTRODUCTION

The state-of-the-art approach to high-volume manufacturing assembly typically makes use of product-dedicated, custom-designed hardware fixtures. For example, in the current assembly of sheet metal parts to produce automotive bodies, clamping fixtures are used to accurately position, orient, and immobilize each sheet metal piece prior to robot welding. A typical automotive body is composed of 300–500 different pieces of sheet metal that must be assembled; as a consequence, a great number of costly fixtures are required. The cost of fixtures ranges between...
$100–200 million (Canadian) per assembly plant per year, to accommodate the annual introduction of new or modified automobile body designs,\(^2\) In addition, these fixtures, which require four to six months\(^3\) to design and manufacture, are not designed to be reconfigurable or reusable, thus reducing manufacturing flexibility.

The underlying motivation for this work is then the development of a new assembly technology, called flexible fixtureless assembly,\(^2\) to eliminate dedicated hardware fixtures. Fixtureless assembly is based on the replacement of fixtures by reprogrammable robotic manipulators and sensors. For example, a typical fixtureless assembly may consist of the following operations: two robots each grasp one part, correctly position and orient the parts for mating, and then a third robot is used to weld the two parts, forming an assembly. Assemblies composed of more than two sheet metal pieces are assembled through repeated applications of this process. However, to reduce the cost of this process, parts are presented to the robots by low-accuracy part holders. Approximate initial locations of the parts are known: the uncertainty in the location of the parts, when placed in a part holder, is 5–10 percent of part dimensions. Grasping ”mislocated” parts results in misalignment between the parts when they are brought together by the two robots, and renders the subsequent bonding operation by a robot welder unsuccessful. For successful assembly, the parts, once grasped, must be localized in both position and orientation, within required tolerances for assembly. The assembly tolerances are substantially less than those of the part holder.

The localization of a sheet metal part is a six-degrees-of-freedom (6-DOF) problem; three mutually perpendicular translations and their corresponding rotations must be determined. Automotive body dimension tolerances require that each sheet metal part be positioned within a positional accuracy of ±0.5 mm. The corresponding orientational accuracy can be obtained by assuming that the edges of the part can be displaced up to the maximum positional specification of ±0.5 mm. To address the question of which measurements are required to localize the part in three dimensions, a suitable sensing method must be developed to detect distinctive and complex geometrical features of the sheet metal part, that is, thin, irregular edges and curved surfaces. The selection and distribution of measurement points, which affect the accuracy of localization, is also discussed.

The purpose of this article is to present a near-practice demonstration of the use of robotic technology, in place of fixtures, for assembly. This article represents the first reported work that addresses the three-dimensional (3-D) localization of arbitrarily shaped thin-walled sheet metal parts grasped by robots. Though the localization system developed in this work is specific to automotive sheet metal parts, the results may be applied to other manufacturing and assembly tasks. Note that as a first step in ongoing research on fixtureless assembly, the localization system setup described here involves a single robot. The contribution of this work is the demonstration of the proposed localization methodology using an experimental test bed. In the development of the experimental test bed, we present a novel sensing system that is particularly effective in measuring 3-D feature geometry (i.e., thin edges) of sheet metal parts by combining surface and edge observations.

The organization of this article is as follows. In Section 2, literature relevant to this work is reviewed. In Section 3, the general 3-D localization problem of sheet metal parts is described in detail. In Section 4, the localization problem as a direct calibration problem\(^9\) is formulated: (1) find a correspondence between deviations in part location and corresponding variations in sensor measurements, based on the singular value decomposition (SVD) formulation, and (2) incorporate the correspondence into an on-line, iterative localization procedure. A brief overview of the proof-of-concept test bed system is presented in Section 5, which is followed in Section 6 by experimental results using the test bed system. In Section 7, an evaluation and discussion of the experimental results is presented, and the question of robot repeatability error is pursued in an Appendix.

## 2. LITERATURE REVIEW

In this section, we briefly review the literature pertinent to part localization in robotic assembly. Over the preceding decade, due to the need to automate various manufacturing processes, part localization has received considerable attention by researchers working in the area of machine vision. A majority of publications in this area address both part recognition and localization (i.e., refs. 5 and 6). These papers address a problem in which a part must be recognized before it is localized. However, in our work, the part to be localized is already known. Therefore, the objective of this work focuses only on localization of the part.

A major component of our work is devoted to the development of an effective localization sensing technique. A research area of relevance to this problem is that of visual sensing systems for localization.
of parts in assembly or manufacturing. Many published papers pertain to this area; however, few of the vision systems reported thus far are applicable to the problem considered here, because they have been developed for localization of objects of simple shape. For example, ref. 7 proposes a stereo vision system, which is composed of two cameras and an ultrasonic proximity sensor, for detecting and localizing paper objects; ref. 8 uses a tactile sensing system to localize a cube; refs. 6 and 9 employ scanning beam sensors and multiple cameras, respectively, to localize 2-D planar objects; and ref. 10 proposes a sensor system composed of a camera and a series of mirrors to measure 3-D misalignment in cylindrical peg-in-hole insertion.

Another significant area relevant to our work is the development of localization methods. Herein, we discuss published works that assume that the approximate location of a part is known. As in our work, these deal with a localization problem in which the uncertainty in the location of the part is small relative to the dimensions of the part, but much greater than the tolerances allowed by the assembly or manufacturing operations. The problem is typically modeled and solved by matching a database surface description of the part with discrete points measured on its surface. This is typically accomplished through minimization of a least-squares function. Two general strategies have arisen to create the part description database: the CAD model method11–13 and the direct calibration method.9 The CAD model method is achieved by matching a CAD database description of the part surface with points measured on the actual surface. In refs. 11 and 12, the general 3-D part localization problem is studied using the CAD model method, and simulation results are presented. This approach represents the most common method for localizing parts in manufacturing environments. In contrast, the direct calibration method does not require a CAD model; instead, it uses best-fit mappings that describe the direct correspondence between part location and sensor measurements. In ref. 9, theoretical and simulation results that consider the localization of 2-D planar parts are presented.

The accuracy achieved by localization methods is largely dependent on how accurately the part description database represents the actual part. The introduction of programmable numerical control techniques in the auto industry has allowed dimensionally accurate cutting and forming of sheet metal parts from CAD models. Hence, no matter which method of cutting or forming is used, the dimensional variations between an actual sheet metal part and its CAD model (or between parts) are typically small (on the order of 0.1 mm).14 In localization during assembly, however, when the CAD model is used as the part description database to represent the actual sheet metal part, the geometric errors between the CAD model and the actual part could be relatively large (i.e., >0.5 mm). This is due to the high structural flexibility of sheet metal parts, which allows the actual part grasped by a robot to deform under gravity and grasping forces.15 In contrast, the part description database obtained through direct calibration can inherently account for these small geometrical changes by determining the best-fit mappings of the relationships between the measurement points and the part locations. This is accomplished on-line, at a specified location where the part is to be mated with another. This article extends the direct calibration localization method to the more general case: 3-D localization of a thin-walled sheet metal part of arbitrary shape.

3. PROBLEM DESCRIPTION

Consider a sheet metal part, which is placed into a crude part holder. With this part holder, the part is arbitrarily placed, close (within 5–10 percent of the dimension of the part) to the correct location, within the robot workspace. The robot will attempt to grasp the part at the same programmed location in the part holder every time; however, since the part is not located precisely, the grasped part cannot be mated properly with other parts.

Figure 1 is a schematic of a robot grasping a part. The solid and dashed lines show the actual (mislocated) and the desired location of the part, respectively. The incorporation of an automatic part localization system allows the robot to correct the mislocated part to the desired position and orientation for subsequent mating or welding operations. Using sensors, the part localization system determines the amount of the mislocation of the grasped part with respect to a reference coordinate frame, that is, the desired location (see Fig. 1). With these data, compensation for the part mislocation can be achieved.

We summarize the underlying assumptions used in our work:

A.1: The size and shape of the part to be localized is known.
A.2: The desired location of the part in the workspace is known.
A3: The mislocation of the part is small relative to the dimensions of the part, but more than an order of magnitude greater than the tolerances allowed.

Assumption A.3 is easily satisfied by the employment of the low-accuracy part holder.

Figure 1 illustrates the part coordinate frame A-xyz, attached to the part, and the reference coordinate frame D-xyz, which represents the desired location of the part. The mislocation of the part is defined as the location of the frame A-xyz relative to the frame D-xyz. Let \((w_p, p_p, r_p)\) denote respectively the yaw, pitch, and roll angles of the part orientation, and \((x_p, y_p, z_p)\) the part position relative to the frame D-xyz, as shown in Figure 2. We denote by \(\Delta L \in \mathbb{R}^{1 \times 6}\) the mislocation vector that describes the complete 6-DOF position and orientation of the part, that is,

\[
\Delta L = [x_p\ y_p\ z_p\ w_p\ p_p\ r_p]
\]

Hence, \(-\Delta L\) is the amount by which the robot end-effector must move to compensate for the mislocation. Note that \(\Delta L\) represents the location of the frame A-xyz with respect to the frame D-xyz, where the location of the frame D-xyz is already known (from Assumption A.2).

The mislocation of the part, \(\Delta L\), is determined through the use of a number of spatially distributed sensors. For sufficiently accurate 6-DOF localization of thin-walled sheet metal parts, both thin edges and surfaces must be detected. We employed a set of seven laser sensors, each providing a single measurement point on the part: three points on the top surface (using proximity sensors) and four points around the edges (using edge sensors). The sensors were placed in a configuration such that the proximity and edge sensors measured the distances to the part in the \((z_p, w_p, p_p)\) surface-related coordinates and the \((x_p, y_p, r_p)\) edge-related coordinates, respectively (see Fig. 2). Now, let \(S \in \mathbb{R}^{1 \times 7}\) be defined as the sensor measurement vector

\[
S = [s_1\ s_2\ s_3\ s_4\ s_5\ s_6\ s_7]
\]

where \(s_i, i = 1, \ldots, 4\), correspond to the outputs from proximity sensors, and \(s_i, i = 5, \ldots, 7\), to those from edge sensors, expressed with respect to each sensor’s local coordinate frame.

Figure 2 illustrates the relative placement of each measurement point and sensor coordinate frame with respect to the part. Note that the measurement points are widely distributed over the surface and the edges. Sensor \(s_7\) is a redundant sensor, in order to achieve better localization accuracy.

4. PROBLEM FORMULATION

In this section, the formulation of the localization algorithm for 3-D thin-walled sheet metal parts is presented. The algorithm determines the location of the part with respect to the frame D-xyz, shown in Figure 2. The algorithm is based on the direct calibration method,\(^9\) which is composed of two procedures: (1) computation of the correspondences (henceforth referred to as “mappings”) between the part
mislocations $\Delta L$ and sensor measurements $S$, and (2) on-line localization based on the computed mappings.

### 4.1. Mapping Procedure

The mapping procedure is performed off-line. It is initiated by acquiring sensor output data and generating part mislocations, necessary to compute the mapping. Note that these data are obtained while the part is grasped by the robot. Then, from these mapping data, the best-fit mapping between the part mislocations and the sensor measurements is computed.

**Step 1. Acquisition of Mapping Data**

First, define the desired location $(D-xyz)$ of the part with respect to the world frame $W-xyz$ (as described in the previous section, with reference to the coordinate frames shown in Figure 1). Adjust the placement of the sensors, such that a part at the desired location falls well within the measurable range of the sensors. Then, perturb the location of the part by known amounts from the desired location, and take set of measurements at each selected location. Generate a set of $n$ measurements of the part such that they evenly span the region around the desired location, covering the possible mislocation range. Let $\Delta L_i \in \mathbb{R}^{1 \times 6}$ ($i = 1, \ldots, n$) be defined as a vector of the perturbed location (that is, mislocation), with its components identical to that in Eq. (1). Also, referring to Eq. (2), let $S_i \in \mathbb{R}^{1 \times 7}$ ($i = 1, \ldots, n$) be the corresponding perturbed measurement vector. Then, define a set of mapping data, $\Delta L_M \in \mathbb{R}^{n \times 6}$ and $S_M \in \mathbb{R}^{n \times 7}$, as

$$
\Delta L_M = \begin{bmatrix}
\Delta L_1 \\
\vdots \\
\Delta L_n
\end{bmatrix} \quad \text{and} \quad S_M = \begin{bmatrix}
S_1 \\
\vdots \\
S_n
\end{bmatrix}
$$

**Step 2. Determination of Mapping**

In this step, the best-fit mapping between $\Delta L$ and $S$ is established using the mapping data $\Delta L_M$ and $S_M$. Two process mappings are formulated from the preceding set of mapping data: (1) a forward mapping process, which maps the measurement data to the part mislocation data, and (2) an inverse mapping process, which maps the mislocation data to the measurement data. Then, applying a second-order polynomial multiple regression model, the forward and inverse mapping processes can be estimated. Define the inverse mapping process as

$$
\Delta L_M = \tilde{S}_M M_I
$$

where $\tilde{S}_M \in \mathbb{R}^{n \times 6}$ is called the inverse mapping matrix, and $\tilde{S}_M \in \mathbb{R}^{n \times 36}$ is a set of augmented measurement vectors given by

$$
\tilde{S}_M = \begin{bmatrix}
\tilde{S}_1 \\
\vdots \\
\tilde{S}_n
\end{bmatrix}
$$

where

$$
\tilde{S}_i = [1 \ s_1 \ s_1^2 \ s_2 \ s_2^2 \ s_3 \ s_3^2 \ s_4 \ s_4^2 \ s_5 \ s_5^2 \ s_6 \ s_6^2 \ s_7 \ s_7^2]
$$

for $i = 1, \ldots, n$. Similarly, the forward mapping process can be estimated by defining

$$
S_M = \Delta L_M M_F
$$

where $M_F \in \mathbb{R}^{28 \times 7}$ is the forward mapping matrix, and $L_M \in \mathbb{R}^{n \times 28}$ is a set of augmented location vectors given by

$$
\Delta L_M = \begin{bmatrix}
\Delta L_1 \\
\vdots \\
\Delta L_n
\end{bmatrix}
$$

where

$$
\Delta L_i = [1 \ x_p \ \cdots \ r_p \ x_p^2 \ x_p y_p \ \cdots \ x_p r_p \ y_p^2 \ y_p z_p \ \cdots \ r_p^2]
$$

for $i = 1, \ldots, n$. When $n > 7$, Eqs. (4) and (7) represent an overdetermined set of linear equations that can be solved for $M_I$ and $M_F$, respectively, using the SVD method for nonsquare matrices.\textsuperscript{17}

### 4.2. Localization Procedure

The localization procedure is performed in real time, as the robot moves through a simulated part-mating process. During the localization of an arbitrarily placed part, the part mislocation $\Delta L$ is determined from the sensor measurements $S$ using the two mapping matrices, $M_I$ and $M_F$, computed from the mapping process. Here, we outline a two-step iterative localization algorithm from ref. 9.

**Step 3. Estimation of Initial Mislocation**

Once an arbitrarily placed part is grasped by the robot from the part holder, sensors take a set of measurements to construct $S$, the measurement vector, given by
Step 4. Iterative Improvement of Estimated Mislocation

This step is required for further improvement of the initial estimate of the mislocation vector obtained in Step 3. Using the forward mapping process, from Eq. (6), define

\[ \hat{\Delta}L = \hat{S}M_f \]  

where

\[ \hat{\Delta}L \in \Re^{1 \times 6} \] = initial estimate of the mislocation vector \( \Delta L \)
\[ \hat{S} \in \Re^{1 \times 36} \] = augmented measurement vector of \( S \)

in which the components of \( \hat{S} \) are defined as in Eq. (6).

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\[ \hat{\Delta}L = \hat{S}M_f \]  

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\[ \hat{S} \in \Re^{1 \times 36} \] = augmented measurement vector of \( S \)

in which the components of \( \hat{S} \) are defined as in Eq. (6). Now, define the squared error between the estimate \( \hat{S} \) and the sensor measurements \( S \) as

\[ F = \frac{1}{2} \sum_{i=1}^{7} \lambda_i (\hat{s}_i - s_i)^2 \]  

where the \( \lambda_i \) are scalar sensitivity weights for the sensors, and \( \hat{s}_i \) and \( s_i \) are the components of \( \hat{S} \) and \( S \), respectively. Compute the gradient of \( F \) as

\[ \nabla F = \begin{bmatrix} \sum_{i=1}^{7} \lambda_i (\hat{s}_i - s_i) \left( \frac{\partial \hat{S}_1}{\partial x_p} \right) \\ \vdots \\ \sum_{i=1}^{7} \lambda_i (\hat{s}_i - s_i) \left( \frac{\partial \hat{S}_7}{\partial x_p} \right) \end{bmatrix}, \quad \nabla F \in \Re^{6 \times 1} \]  

where \( \frac{\partial \hat{S}_i}{\partial x_p} \sim \frac{\partial \hat{S}_i}{\partial \tilde{f}_p}, \) \( i = 1, \ldots, 7 \), are obtained by computing the gradient \( \hat{S} \) from Eq. (11). Then, using \( \nabla F \), a correction to the mislocation estimate is obtained as

\[ \hat{\Delta}L_{k+1} = \hat{\Delta}L_k - \nabla F \ell \]  

where

\[ \hat{\Delta}L_k \in \Re^{1 \times 6} \] = initial estimate of the mislocation vector \( \Delta L \)
\[ \hat{\Delta}L_{k+1} \in \Re^{1 \times 6} \] = updated estimate of the mislocation vector \( \Delta L \)
\[ \ell \] = scalar step length parameter, set to 0.01

We iterate \( k = 1, 2, \ldots \) by substituting \( \hat{\Delta}L_{k+1} \) back into Eq. (11) until \( F \) is less than some predefined value or \( k \) reaches a maximum number of iterations. The algorithm to estimate the mislocation \( \Delta L \) is summarized in Figure 3.

**Figure 3.** Localization algorithm.

5. TEST BED LOCALIZATION SYSTEM

In order to demonstrate the feasibility of the proposed localization methodology, a test bed localization system was developed. A schematic diagram of the system is shown in Figure 4. The test bed utilized a single 6-DOF FANUC S-110 robot, which has maximum load capacity of 10 kg and position repeatability error of ±0.2 mm. The robot was controlled by a FANUC KAREL R-F controller, which was interfaced to a host computer via an RS232 serial connection.

A vacuum gripper with multiple grip points was designed to support the surface of a single automotive front quarter panel, and simultaneously (and passively) damp out vibration. The gripper was composed of three rubber suction cups with metal-detecting proximity switches, mounted on an aluminum frame structure, and a vacuum generator (ISI Automation Inc. VMS-2110), which created a vacuum for the suction cups.
The host computer was a Pentium 166 MHz PC. Communication between the PC and the sensors used in the system was controlled by Visual Designer software running in the host computer, interfaced with an external data acquisition board (Daytronic 10KU-KD). The part localization algorithms were programmed in MATLAB. The main command and communication module, which directed the overall localization process, was implemented in C++.

The sensor system was composed of seven laser-based sensors: three proximity (NAiS LM100) and four edge (NAiS UZL100) sensors. The proximity sensors were diffuse-reflective beam sensors, that is, the emitter and receiver were coaxially placed (see Fig. 5(a)). (In this type of sensor, when an emitted laser beam makes contact with the sheet metal part, some of the diffuse-reflected beam returns to its receiver.) As discussed in Section 3, the proximity sensors detect the position of the surface of the part. The stand-off range of the sensor is 50 mm, with a resolution of 0.02 mm or less, and a response time of 0.04 m.

The edge sensors were through-beam type, that is, the emitter and receiver were positioned opposite to each other (see Fig. 5(b)). When a part passed between them, a portion of the emitted beam was blocked, and only the unblocked rays reached the receiver. The edge sensors detect the edges of the part. The stand-off range is 15 mm, with a resolution of 0.01 mm or less and a response time of 0.5 ms. Both types of sensors exhibit material-related measurement error, which affects localization performance. (Note that this error is related to the surface roughness and glossiness of the part. For example, for a steel plate, the material-related error is specified at approximately 0.1 mm.)

6. EXPERIMENTAL RESULTS

In this section, we discuss the performance of the proposed localization methodology determined from experiments conducted using the test bed system. An automotive front quarter panel was used as the part in testing the localization system, as shown in Figure 6. (The dimensions of the quarter panel were shown in Figure 2.) Recall from Section 1 that the automotive body assembly tolerance requires the quarter panel to be positioned within an accuracy of ±0.5 mm, and, the corresponding orientation accuracy is calculated to be ±0.07°.
6.1. Mapping Procedure

To compute the mapping matrices, $M_I$ and $M_F$, an arbitrary total of $n = 729$ perturbation points were formed, by taking all combinations of the following perturbations: 3 points at 2 mm intervals for each of the $x_p$, $y_p$, and $z_p$ position coordinates, and 3 points at $0.5^\circ$ intervals for each of the $w_p$, $p_p$, and $r_p$ orientation coordinates, as given in Eq. (1). These combinations evenly span the quarter panel’s possible mislocation range, $\pm 2$ mm in each position coordinate and $\pm 0.5^\circ$ in each orientation coordinate. This range was only limited by the stand-off range (15 mm) of the edge sensors and the size of the quarter panel (as shown in Figure 2). As previously discussed, perturbations to the grasped quarter panel location were introduced by moving the robot end-effector by known amounts about the desired location. Note that the time required to automatically give the part a preprogrammed perturbation and save the corresponding sensor readings was about 1.5 seconds. Hence, the overall mapping procedure took less than 20 minutes.

6.2. Localization Procedure: Performance Results

The localization procedure was performed an arbitrary total of 129 times, to accumulate statistics to determine the repeatability and performance of the proposed methodology. In these trials, the predefined maximum number of iterations ($k$) for estimating the part mislocation was chosen to be 500, and the sensor sensitivity weights ($\lambda_i$) in Eq. (12) were set to unity. Intentional part mislocations were introduced by jogging the robot, while the quarter panel was in its grasp, by known amounts. Using the localization algorithm described in Section 4, the estimated mislocation of the grasped quarter panel, $\Delta \hat{L}$, was calculated. Then, the robot end-effector was jogged by $-\Delta \hat{L}$ with respect to the current location to correct the mislocation. The performance error statistics of 129 trials are given in Table I. For each trial, the localization error is the difference between the estimated and the actual mislocation.

The results shown in the table demonstrate that the mean values for the $x_p$, $y_p$, and $z_p$ coordinate errors are well within the target tolerance value of $\pm 0.5$ mm. The mean errors for the $w_p$, $p_p$, and $r_p$ orientation coordinates are also within the target value of $\pm 0.07^\circ$. However, the nonzero mean values indicate that there are systematic shifts in estimation of mislocation. The maximum and minimum errors do not exceed the target values, except in the $w_p$ coordinate where the minimum error is slightly outside the target range. Figure 7 uses plots to illustrate sample localization errors from $x_p$ to $r_p$ coordinates. Each point on a plot represents a single trial, whereas the dashed lines show the target tolerance bounds. The positional and orientational estimates were repeatable to $0.1$ mm and $0.02^\circ$, respectively.

It was observed from experiments that the localization error associated with each coordinate is a linear function of its own mislocation. For example, the degree of $x_p$ error is linearly dependent on the degree of $x_p$ mislocation. Hence, when there are no systematic shifts, the measurement error increases as its coordinate mislocation increases. However, for a complete 6-DOF localization, the mislocation of one coordinate also affects the localization errors of all the other coordinates. As a consequence, the error values are difficult to predict and generally show no trend (see Fig. 7), and it becomes difficult to perform sensitivity analysis between the measurement errors and mislocations.

The system showed satisfactory performance in terms of computational efficiency. The computation time to estimate part mislocation and then perform 500 iterations to refine the part mislocation was approximately two seconds, and an additional one
second was required for the robot to physically correct the computed mislocation. The decrease in localization error during iterations is dictated by $\nabla F$ (see Eq. (13)). The behavior of $\nabla F$ is discussed in more detail in ref. 9. By trial and error, it was found that 500 iterations gave the best results (i.e., smallest localization errors).

7. DISCUSSION

Our results confirm that the proposed methodology can be used to localize thin-walled sheet metal parts to within target tolerances. It is well known that localization performance is governed by the accuracy of the sensing system. We expect the localization accuracy to be of the same order as the combined average of the material-related measurement error (as discussed in Section 5) and sensor noise. Note that the sensors employed have resolution of the order of 0.01 mm; however, the material-related measurement error of a steel sheet could be as large as 0.1 mm. Also, the localization performance is dependent on the accuracy of the mappings, given by Eqs. (4) and (7), which were obtained from a limited data set. Hence, the performance can also be improved by increasing the number of perturbation points in the mapping sets. In addition, the localization performance is strongly related to the number of sensors used. A reduced number of sensor measurements leads to a greater localization error. Evidence of this can be seen in Table I, where the localization performance of the edge-related coordinates $(x_p, y_p, r_p)$ is slightly better than that of the surface-related coordinates $(z_p, w_p, p_p)$. The reason for this is that the edge coordinates are primarily detected by four edge sensors, whereas the surface coordinates are detected largely by three proximity sensors. The selection of measurement points is also important. The sensors should be directed towards less curved points on the geometry of the part for better computational accuracy. The measurement points also should be widely spread and evenly distributed over the surface and around the edges of the part, for better estimation of part position and orientation.

Another source of error is the repeatability error of the FANUC S-110 robot, which has a variance as large as $\pm 0.2$ mm (in position coordinates), as indicated in Section 5. Note that in the mapping procedure, the
sampled perturbation points in part location were introduced by the robot, and thus were subject to its repeatability error (note also that the repeatability error is one order of magnitude smaller than the perturbations). Obviously, the use of a different number \((n)\) of perturbation points will produce different mapping matrices. The question then arises whether the increase in the number of perturbation points will reduce the effect of the repeatability error, and thus result in more accurate representation of the true mapping matrices. This question is pursued in the Appendix, where analysis shows that the location of the part can be estimated with an accuracy greater than its repeatability error. It is reasonable to assume that the location of the part more accurately than its repeatability error. Nonetheless, the allowable localization accuracy required for the assembly task in this work (i.e., ±0.5 mm in position coordinates) is greater than the repeatability error, and the results presented here are a good indicator that the proposed localization system works.

**APPENDIX**

**Effect of Robot Repeatability Error on Localization**

Let us consider the case of the inverse mapping process, described by Eq. (4), as an example. Suppose that \(\Delta L_M\) is the mislocation uncertainty matrix due solely to the robot repeatability error, and ignore the measurement error due to the sensors (as it is much smaller than the repeatability error). It is reasonable to assume that the repeatability error of the robot possesses a normal distribution.\(^{19}\) Define \(\tilde{M}_I\) as the inverse mapping uncertainty matrix due to the mislocation uncertainty. Then, Eq. (4) can be rewritten as

\[
(\Delta L_M + \Delta \tilde{L}_M) = \tilde{S}_M (M_I + \tilde{M}_I)
\]  

(A.1)

where \(\Delta L_M, \tilde{S}_M,\) and \(M_I\) are as defined in Section 4.1. Using a probabilistic approach, the covariance matrix of the inverse mapping matrix is given by

\[
E\{\tilde{M}_I \tilde{M}_I^T\} = \tilde{S}_M \Delta L_M \Delta \tilde{L}_M^T (\tilde{S}_M^T)^{-1} \quad (A.2)
\]

Without loss of generality, assume that the mislocation uncertainty due to the repeatability error is uncorrelated and has identical variances \(\sigma^2_{\Delta L}\) in all coordinates, so that \(R = \sigma^2_{\Delta L} I\). Then, Eq. (A.2) reduces to

\[
E\{\tilde{M}_I \tilde{M}_I^T\} = (\tilde{S}_M^T \tilde{S}_M)^{-1} R  
\]  

(A.3)

For the number of sample perturbation points \((n = 729)\) used in this work, \((\tilde{S}_M^T \tilde{S}_M)^{-1}\) in Eq. (A.3) is very small and thus leads to very small variance in the inverse mapping. However, note that the dimensions of \((\tilde{S}_M^T \tilde{S}_M)^{-1}\) are too large to give the matrix here.

Additionally, the mislocation uncertainty \(\Delta L_M\) for a given sensor measurement vector \(S\) can be evaluated quantitatively using Eqs. (10) and (A.3). The covariance matrix of the mislocation uncertainty due to the robot repeatability error is given by

\[
E\{\Delta \tilde{L}_M \Delta \tilde{L}_M^T\} = \tilde{S}_M \tilde{M}_I \tilde{S}_I^T = \tilde{S}_M^T \tilde{S}_M^{-1} \tilde{S}_I^T R  
\]  

(A.4)

Using Eq. (A.4) and a sample sensor measurement vector, \(E(\Delta L_M \Delta L_M^T) = 0.0181R\) was obtained for \(n = 729\). This implies that the mislocation vector \(\Delta L_M\) can be determined with a covariance that is 0.0181 times the covariance of the repeatability of the robot. Also, note that this result is substantially better than that for, a set of data for \(n = 250\), in which \(E(\Delta L_M \Delta L_M^T) = 6.841R\). This result clearly demonstrates that the effect of the robot repeatability error on localization can be minimized by increasing the number of sample perturbation points.

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**REFERENCES**